Integral Notes

1. $\int a\,dx = ax + c$, where a is a constant

2. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c$, where $n \neq -1$

3. $\int \dfrac{1}{x}\,dx = \ln|x| + c$

4. $\int a^x\,dx = \dfrac{a^x}{\ln(a)} + c$

5. $\int e^x\,dx = e^x + c$

6. $\int \ln(x)\,dx = x\ln(x) - x + c$

7. $\int \sin(x)\,dx = -\cos(x) + c$

8. $\int \cos(x)\,dx = \sin(x) + c$

9. $\int \sec^2(x)\,dx = \tan(x) + c$

10. $\int cf(x)\,dx = c\int f(x)\,dx$, where c is a constant

11. $\int (f \pm g)\,dx = \int f\,dx \pm \int g\,dx$

Integration

12. by Parts

- Is often useful when 2 functions are multiplied together.

- $\int uv\,dx = u\int v\,dx - \int u'\left(\int v\,dx\right)dx$

- E.g. 1 $\int x\cos(x)\,dx$

$u = x$
$v = \cos(x)$

$u\int v\,dx - \int u'\left(\int v\,dx\right)dx$
$= x\int \cos(x)\,dx - \int x'\left(\int \cos(x)\,dx\right)dx$
$= x\sin(x) - \int \sin\,dx$
$= x\sin(x) + \cos(x) + c$

- E.g. 2 $\int \ln(x)\,dx$

$u = \ln(x)$
$v = 1$

$u\int v\,dx - \int u'\left(\int v\,dx\right)dx$
$= \ln(x)\int 1\,dx - \int (\ln(x))'\left(\int 1\,dx\right)dx$
$= x\ln(x) - \int \left(\frac{1}{x}\right)(x)\,dx$
$= x\ln(x) - \int 1\,dx$
$= x\ln(x) - x + c$

- E.g. 3 $\int \frac{\ln(x)}{x^2}\, dx$

$u = \ln(x)$

$v = \frac{1}{x^2}$

$$u \int v\, dx - \int u' \left(\int v\, dx \right) dx$$

$$= \ln(x) \int x^{-2}\, dx - \int (\ln(x))' \left(\int x^{-2}\, dx \right) dx$$

$$= \frac{-\ln(x)}{x} - \int \left(\frac{1}{x}\right)\left(\frac{-1}{x}\right) dx$$

$$= \frac{-\ln(x)}{x} + \int \frac{1}{x^2}\, dx$$

$$= \frac{-\ln(x)}{x} - \frac{1}{x} + c$$

$$= -\left(\frac{\ln(x)+1}{x} \right) + c$$

- E.g. 4 $\int e^x x\, dx$

$u = x$

$v = e^x$

$$u \int v\, dx - \int u' \left(\int v\, dx \right) dx$$

$$= x \int e^x\, dx - \int x' \left(\int e^x\, dx \right) dx$$

$$= xe^x - \int e^x\, dx$$

$$= xe^x - e^x + c$$

$$= e^x (x-1) + c$$

Note: If you chose e^x as u and x as v, you would have had a messy equation.

We need to choose u and v carefully. We should choose a u that gets simpler when you differentiate it and a v that doesn't get any more complicated when you integrate it.

As a general rule, choose u based on which of these comes first:

1. Inverse trig functions
2. Log functions
3. Algebraic functions
4. Trig functions
5. Exponential functions

Integration By Substitution

- The first and most vital step is to write the integral in this form:

$$\int f(g(x))\, g'(x)\, dx$$

- When the integration is set up like above, we can do:

$$\int \underbrace{f(g(x))} \; \underbrace{g'(x)\,dx}$$

$$\int f(u) \quad du$$

- E.g. 1 $\int \cos(x^2) \cdot 2x \, dx$

let $u = x^2$

$\frac{du}{dx} = 2x \longrightarrow dx = \frac{du}{2x}$

$\int \cos(u)\, du$

$= \sin(u)$

$= \sin(x^2) + c$

- E.g. 2 $\int \cos(x^2)\, 6x\, dx$

$= 3\int \cos(x^2)\, 2x\, dx$

$= 3\sin(x^2) + c$

- E.g. 3 $\int \frac{x}{x^2+1}\, dx$

Let $u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{1}{2u}\, du$

$= \frac{1}{2}\int \frac{1}{u}\, du$

$= \frac{1}{2}(\ln|u|)$

$= \frac{1}{2}(\ln(x^2+1)) + c$